Simple Harmonic Motion

Question1

If the function $\sin^2 \omega t$ (t is time in second) represents a periodic motion, then the period of the motion is

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

$\sqrt{\frac{\pi}{\omega}} \mathrm{s}$

B.

$\frac{\pi}{\omega} \mathrm{s}$

C.

$\frac{2\pi}{\omega} s$

D.

$\sqrt{\frac{2\pi}{\omega}} \mathrm{s}$

Answer: B

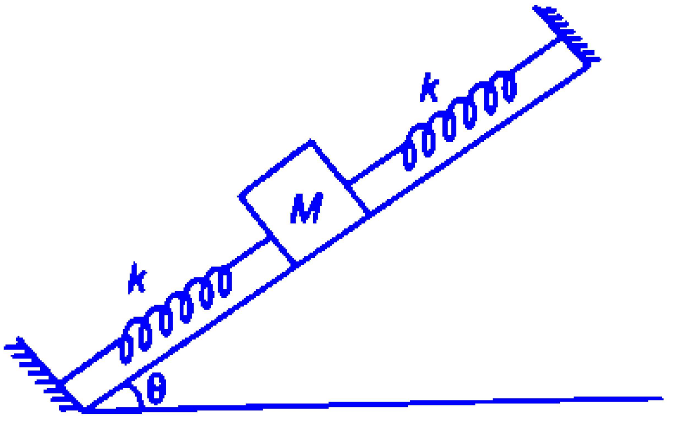
Solution:

$$\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2} = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$$
$$\therefore \quad T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega} \mathrm{s}$$

Question2

On a smooth inclined plane a block of mass M is fixed to two rigid supports using two springs as shown in the figure. If each spring has spring constant k, then the period of oscillation of the block is

(Neglect the masses of the springs)



AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

$2\pi\left(\frac{M}{2k}\right)^{1/2}$

B.

$2\pi\left(\frac{2M}{k}\right)^{1/2}$

C.

$2\pi\left(\frac{Mg\sin\theta}{2k}\right)^{1/2}$

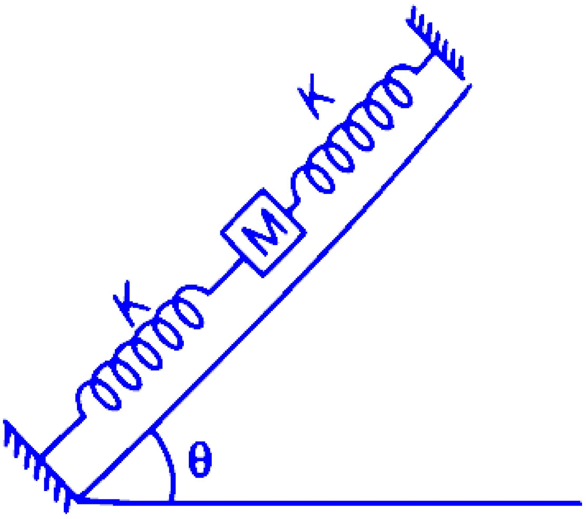
D.

$2\pi\left(\frac{2Mg}{k}\right)^{1/2}$

Answer: A

Solution:

According to given figure, springs are connected in parallel.



Therefore effective spring constant

$$K_{\text{eff}} = K_1 + K_2$$
$$= K + K = 2K$$

The restoring force on block due to springs not due to gravity pull

So, $T = 2\pi\sqrt{\frac{M}{K_{\text{eff}}}} \Rightarrow T = 2\pi\sqrt{\frac{M}{2K}}$

Question3

A particle is executing simple harmonic motion with amplitude A. The ratio of the kinetic energies of the particle when it is at displacements of $\frac{A}{4}$ and $\frac{A}{2}$ from the mean position is

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

4 : 1

B.

2 : 1

C.

5 : 4

D.

9 : 16

Answer: C

Solution:

$$\begin{aligned}\frac{k_1}{k_2} &= \frac{\frac{1}{2} m\omega^2\left(A^2-x_1^2\right)}{\frac{1}{2} m\omega^2\left(A^2-x_2^2\right)} \\ &= \frac{A^2-x_1^2}{A^2-x_2^2}=\frac{A^2-\left(\frac{A}{4}\right)^2}{A^2-\left(\frac{A}{2}\right)^2} \\ &= \frac{\frac{15}{16} A^2}{\frac{3}{4} A^2}=\frac{5}{4}\end{aligned}$$

Question4

A particle is executing simple harmonic motion starting from its mean position. If the time period of the particle is 1.5 s , then the minimum time at which the ratio of the kinetic and total energies of the particle becomes 3:4 is

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

$\frac{1}{4}$ s

B.

$\frac{1}{12}$ s

C.

$\frac{1}{8}$ s

D.

$\frac{1}{6}$ s

Answer: C

Solution:

For particle in SHM,

$$T = 1.5 \text{ sso}$$
$$\omega = \frac{2\pi}{T} = \frac{4\pi}{3} \text{rad/s}$$

Let oscillations starts from $t = 0$ and $x = A \sin \omega t$ is displacement equation Then, KE at time t is,

$$\begin{aligned} K &= \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 \\ &= \frac{1}{2} m \cdot A^2 \omega^2 \cos^2 \omega t \qquad \ldots (i) \end{aligned}$$

And total energy of particle is,

$E = \frac{1}{2} k A^2$

Here, $k = m\omega^2$

$= \frac{1}{2} m A^2 \omega^2 \qquad \ldots (ii)$

$$\begin{aligned} &\text{Given, ratio, } \frac{K}{E} = \frac{3}{4} \\ &\Rightarrow \quad \frac{\frac{1}{2} m A^2 \omega^2 \cos^2 \omega t}{\frac{1}{2} m A^2 \omega^2} = \frac{3}{4} \\ &\Rightarrow \quad \cos^2 \omega t = \frac{3}{4} \\ &\Rightarrow \quad \cos \omega t = \sqrt{\frac{3}{2}} \\ &\Rightarrow \quad \omega t = \frac{\pi}{6} \\ &\Rightarrow \quad t = \frac{\pi}{6\omega} = \frac{\pi \times 3}{6 \times 4\pi} = \frac{1}{8} \text{ s} \end{aligned}$$

Question5

The equations for the displacements of two particles in simple harmonic motion are $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$ respectively. The phase difference between the velocities of the two particles at a time $t = 0$ is

Options:

A.

$\frac{\pi}{4}$

B.

$\frac{\pi}{2}$

C.

$\frac{\pi}{6}$

D.

$\frac{\pi}{3}$

Answer: C

Solution:

$$\begin{aligned} y_1 &= 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right) \\ \therefore \quad v_1 &= 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right) \\ &= 10\pi \times \cos\left(100\pi t + \frac{\pi}{3}\right) \\ y_2 &= 0.1 \cos \pi t \\ &= 0.1 \sin\left(\pi t + \frac{\pi}{2}\right) \\ \therefore \quad v_2 &= 0.1 \times \pi \cos\left(\pi t + \frac{\pi}{2}\right) \\ &= 0.1\pi \cos\left(\pi t + \frac{\pi}{2}\right) \end{aligned}$$

$\therefore$ At $t = 0$

$$v_1 = 10 \cos \frac{\pi}{3}$$

and $v_2 = 0.1\pi \cos \frac{\pi}{2}$

$\therefore$ Phase difference between v_1 and v_2 at time $t = 0$,

is given as,

$\phi = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Question6

A spring is stretched by 0.2 m when a mass of 0.5 kg is suspended to it. The time period of the spring when 0.5 kg mass is replaced with a mass of 0.25 kg is suspended to it is

(Acceleration due to gravity $= 10\ \mathrm{ms}^{-2}$ **)**

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

0.628 s

B.

6.28 s

C.

62.8 s

D.

0.0628 s

Answer: A

Solution:

Step 1: Find Spring Constant (k)

The spring stretches by 0.2 m when a 0.5 kg mass hangs from it.

This means the force from the weight ($m_1 g$) equals the force from the spring (kx):

$kx = m_1 g$

So,

$k = \frac{m_1 g}{x} = \frac{0.5 \times 10}{0.2} = 25\ \mathrm{N/m}$

Step 2: Use the New Mass

The new hanging mass is $m_2 = 0.25$ kg.

Step 3: Find the New Time Period (T)

The time period of a mass-spring system is:

$T = 2\pi\sqrt{\frac{m_2}{k}}$

Substitute the values:

$T = 2\pi\sqrt{\frac{0.25}{25}}$

$T = 2\pi\sqrt{0.01}$

$T = 2\pi \times 0.1 = 0.2\pi \text{ s}$

Step 4: Calculate the Value

Using $\pi \approx 3.14$:

$T = 0.2 \times 3.14 = 0.628 \text{ s}$

Question7

A body of mass 4 kg attached to a spring of force constant 64Nm^{-1} executes simple harmonic motion on a frictionless horizontal surface. The time period of oscillation is

AP EAPCET 2025 - 23rd May Morning Shift

Options:

A.

$\frac{\pi}{3}$ s

B.

$\frac{\pi}{2}$ s

C.

πs

D.

$\frac{3\pi}{2}$ s

Answer: B

Solution:

- Mass $m = 4\,\text{kg}$
- Force constant (spring constant) $k = 64\,\text{N/m}$

Formula for time period of a mass–spring system:

$T = 2\pi\sqrt{\frac{m}{k}}$

Substitute the values:

$T = 2\pi\sqrt{\frac{4}{64}} = 2\pi\sqrt{\frac{1}{16}} = 2\pi \times \frac{1}{4} = \frac{\pi}{2}$

✅ **Answer:** $\boxed{\frac{\pi}{2}\ \text{s}}$

Correct Option: B

Question8

A particle is executing simple harmonic motion with amplitude A. At a distance ' x ' from the mean position, when the particle is moving towards extreme position it receives a blow in the direction of motion which instantaneously doubles its velocity. The new amplitude of the particle is

(Frequency is constant during the motion)

AP EAPCET 2025 - 23rd May Morning Shift

Options:

A.

A

B.

$\sqrt{A^2 - X^2}$

C.

$\sqrt{2A^2 - 3x^2}$

D.

$\sqrt{4A^2 - 3x^2}$

Answer: D

Solution:

Let angular frequency of SHM is ω and when is at distance x from mean position, its velocity will be given as

$v = \omega\sqrt{A^2 - x^2} \qquad \ldots (i)$

When its velocity becomes double, let new amplitude is A'.

So, $\quad 2v = \omega\sqrt{A'^2 - x^2}$

Putting the value of v from equation (i)

$$
\begin{aligned}
& 2\omega\sqrt{A^2 - x^2} = \omega\sqrt{A'^2 - x^2} \\
& 2\sqrt{A^2 - x^2} = \sqrt{A'^2 - x^2} \\
\Rightarrow \quad & 4\left(A^2 - x^2\right) = A'^2 - x^2 \\
\Rightarrow \quad & A'^2 = 4A^2 - 3x^2 \Rightarrow A' = \sqrt{4A^2 - 3x^2}
\end{aligned}
$$

Question9

If the displacement ' x ' of a body in motion in terms of time ' t ' is given by $x = A\sin(\omega t + \theta)$, then the minimum time at which the displacement becomes maximum is

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

$\left[\frac{\pi}{2\omega} - \frac{\theta}{\omega}\right]$

B.

$\left[\frac{2\omega}{\pi}-\frac{\omega}{\theta}\right]$

C.

$\left[\frac{\pi}{\omega}-\frac{1}{\omega}\right]$

D.

$\left[\frac{\omega}{\pi}-\frac{\omega}{\pi^2}\right]$

Answer: A

Solution:

$x=A\sin(\omega t+\theta)$

For maximum displacement, $x=A$

$$
\begin{aligned}
& A=A\sin(\omega t+\theta)\\
\Rightarrow\quad & 1=\sin(\omega t+\theta)\\
\Rightarrow\quad & \omega t+\theta=\frac{\pi}{2}\\
\Rightarrow\quad & \omega t=\frac{\pi}{2}-\theta\\
\Rightarrow\quad & t=\frac{\pi}{2\omega}-\frac{\theta}{\omega}
\end{aligned}
$$

Question10

If the maximum velocity and maximum acceleration of a particle executing simple harmonic motion are respectively $5\ \mathrm{ms}^{-1}$ and $10\ \mathrm{ms}^{-2}$, then the time period of oscillation of the particle is

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

πs

B.

2π s

C.

2 s

D.

1 s

Answer: A

Solution:

Given:

- Maximum velocity $v_{\max} = 5\,\text{m/s}$
- Maximum acceleration $a_{\max} = 10\,\text{m/s}^2$

For SHM relationships:

$v_{\max} = \omega A$

$a_{\max} = \omega^2 A$

where ω is the angular frequency, and A is the amplitude.

Step 1: Divide the two equations

$\frac{a_{\max}}{v_{\max}} = \frac{\omega^2 A}{\omega A} = \omega$

$\Rightarrow \omega = \frac{a_{\max}}{v_{\max}} = \frac{10}{5} = 2\,\text{rad/s}$

Step 2: Find the time period

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi\,\text{s}$

✅ **Final Answer:**

$\boxed{T = \pi\,\text{s}}$

Correct Option: A) π s

Question11

A body of mass 1 kg is suspended from a spring of force constant 600Nm^{-1}. Another body of mass 0.5 kg moving vertically upwards

hits the suspended body with a velocity of 3 ms^{-1} and embedded in it. The amplitude of motion is

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

5 cm

B.

15 cm

C.

10 cm

D.

8 cm

Answer: A

Solution:

Combined mass, $m = m_1 + m_2$

$= 1 + 0.5 = 1.5$ kg

Using conservation of momentum,

$$\begin{aligned} & m_1 u_1 + m_2 u_2 = (m_1 + m_2) v \\ \Rightarrow \quad & 1 \times 0 + 0.5 \times 3 = 1.5 v \\ \Rightarrow \quad & v = 1 \mathrm{~m/s} \end{aligned}$$

According to conservation of energy,

$$\begin{aligned} \frac{1}{2} m v^2 &= \frac{1}{2} k A^2 \\ \Rightarrow \quad A &= \sqrt{\frac{m v^2}{k}} \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad \sqrt{\frac{1.5 \times 1^2}{600}} &= \sqrt{\frac{1}{400}} = \frac{1}{20} \mathrm{~m} \\ &= \frac{100}{20} \mathrm{~cm} = 5 \mathrm{~cm} \end{aligned}$$

Question12

For a particle executing simple harmonic motion, the ratio of kinetic and potential energies at a point where displacement is one half of the amplitude is

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

$3:1$

B.

$1:3$

C.

$2:1$

D.

$1:2$

Answer: A

Solution:

$$\begin{aligned} \frac{K}{U} &= \frac{\frac{1}{2} m\omega^2\left(A^2-x^2\right)}{\frac{1}{2} m\omega^2 x^2} \\ &= \frac{A^2-x^2}{x^2} \\ &= \frac{A^2-\left(\frac{A}{2}\right)^2}{\left(\frac{A}{2}\right)^2}\left[\text{Here, } x=\frac{A}{2}\right] \\ &= \frac{\frac{3A^2}{4}}{\frac{A^2}{4}}=\frac{3}{1} \end{aligned}$$

$\therefore K: U=3: 1$

Question13

When the mass attached to a spring is increased from 4 kg to 9 kg , the time period of oscillation increases by 0.2π s. Then, the spring constant of the spring is

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

$80 \mathrm{~N}-\mathrm{m}^{-1}$

B.

$200 \mathrm{~N}-\mathrm{m}^{-1}$

C.

$50 \mathrm{~N}-\mathrm{m}^{-1}$

D.

$100 \mathrm{~N}-\mathrm{m}^{-1}$

Answer: D

Solution:

$$T=2\pi\sqrt{\frac{m}{k}}$$

$$\frac{T_1}{T_2}=\sqrt{\frac{m_1}{m_2}}$$

$$\frac{T}{T+0.2\pi}=\sqrt{\frac{4}{9}}=\frac{2}{3}$$

$$\Rightarrow \quad 3T=2T+0.4\pi$$

$$\Rightarrow \quad T=0.4\pi \text{ s}$$

$\therefore$ Using Eq. (i),

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow \quad 0.4\pi = 2\pi\sqrt{\frac{4}{k}} \Rightarrow 0.2 = \frac{2}{\sqrt{k}}$$

$$\Rightarrow \quad \sqrt{k} = \frac{1}{0.1} \Rightarrow \sqrt{k} = 10$$

$$\Rightarrow \quad k = 100\text{Nm}^{-1}$$

Question14

The kinetic energy of a particle executing simple harmonic motion at a displacement of 3 cm from the mean position is 4 mJ . If the amplitude of the particle is 5 cm , then the maximum force acting on the particle is

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

0.25 N

B.

0.50 N

C.

0.75 N

D.

1.25 N

Answer: A

Solution:

Total energy,

$$E=\frac{1}{2} m \omega^2 A^2$$

$$K+U=\frac{1}{2} m \omega^2(0.05)^2$$

$$\Rightarrow \quad 4 \times 10^{-3}+\frac{1}{2} m \omega^2 x^2=\frac{1}{2} m \omega^2(0.05)^2$$

$$\Rightarrow \quad \frac{1}{2} m \omega^2\left[(0.05)^2-x^2\right]=4 \times 10^{-3}$$

$$\Rightarrow \quad \frac{1}{2} m \omega^2\left[(0.05)^2-(0.03)^2\right]=4 \times 10^{-3}$$

$$\Rightarrow \quad \frac{1}{2} m \omega^2=2.5$$

$$\Rightarrow \quad m \omega^2=5$$

$\therefore$ Maximum force

$F_{\max }=m \omega^2 A=5 \times 0.05=0.25 \mathrm{~N}$

Question15

A body of mass 1 kg is attached to the lower end of a vertically suspended spring of force constant $600 \mathrm{~N}-\mathrm{m}^{-1}$. If another body of mass 0.5 kg moving vertically upward hits the suspended body with a velocity $3 \mathrm{~ms}^{-1}$ and embedded in it, then the frequency of the oscillation is

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

$\frac{5}{\pi} \mathrm{~Hz}$

B.

$\frac{10}{\pi} \mathrm{~Hz}$

C.

$\frac{\pi}{5} \mathrm{~Hz}$

D.

$\pi \mathrm{Hz}$

Answer: B

Solution:

Angular velocity of the system,

$$\omega = \sqrt{\frac{k}{m_{\text{total}}}} = \sqrt{\frac{600}{(1+0.5)}}$$

$$= \sqrt{\frac{600}{1.5}} = 20 \text{rad/s}$$

$$\therefore \quad f = \frac{\omega}{2\pi} = \frac{20}{2\pi} = \frac{10}{\pi} \text{ Hz}$$

Question16

If the displacement y (in cm) of a particle executing simple harmonic motion is given by the equation $y = 5\sin(3\pi t) + 5\sqrt{3}\cos(3\pi t)$, then the amplitude of the particle is

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

5 cm

B.

$5(1+\sqrt{3})$cm

C.

$5\sqrt{3}$ cm

D.

10 cm

Answer: D

Solution:

$$y = 5\sin(3\pi t) + 5\sqrt{3}\cos(3\pi t)$$
$$=10\left[\frac{5}{10}\sin 3\pi t + \frac{5\sqrt{3}}{10}\cos(3\pi t)\right]$$
$$=10\left[\sin 3\pi t \cdot \left(\frac{1}{2}\right) + \cos 3\pi t \cdot \left(\frac{\sqrt{3}}{2}\right)\right]$$
$$=10\left[\sin(3\pi t)\cos\frac{\pi}{3} + \cos 3\pi t \sin\frac{\pi}{3}\right]$$
$$=10\sin\left(3\pi t + \frac{\pi}{3}\right)$$
$$\therefore A = 10 \text{ cm}$$

Question17

The angular frequency of a block of mass 0.1 kg oscillating with the help of a spring of force constant $2.5 \mathrm{~N}-\mathrm{m}^{-1}$ is

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

$02 \mathrm{rads}^{-1}$

B.

$5 \mathrm{rads}^{-1}$

C.

$10 \mathrm{rads}^{-1}$

D.

$2 \mathrm{rads}^{-1}$

Answer: B

Solution:

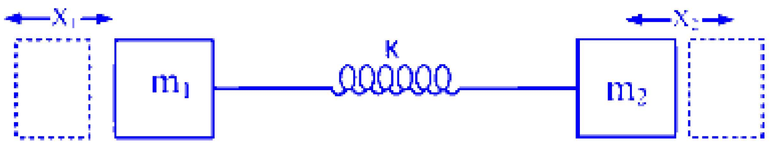
$$n=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{25}{0.1}}$$
$$n=\frac{5}{2\pi}\text{ Hz}$$

$\therefore$ Angular frequency, $\omega=2\pi n$

$=2\pi\times\frac{5}{2\pi}=5\text{rad s}^{-1}$

Question18

As shown in the figure, two blocks of masses m_1 and m_2 are connected to spring of force constant k. The blocks are slightly displaced in opposite directions to x_1, x_2 distances and released. If the system executes simple harmonic motion, then the frequency of oscillation of the system (ω) is



AP EAPCET 2024 - 23th May Morning Shift

Options:

A. $\left(\frac{1}{m_1}+\frac{1}{m_2}\right)k^2$

B. $\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)k^2}$

C. $\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)}$

D. $\sqrt{\left(\frac{1}{m_1}+\frac{1}{m_2}\right)k}$

Answer: A

Solution:

To determine the frequency of oscillation for the system, we need to consider the concept of reduced mass, which is useful when analyzing systems involving two masses connected by a spring.

First, the reduced mass (μ) of a two-body system is given by the following formula:

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

where m_1 and m_2 are the masses of the two blocks connected to the spring. The reduced mass essentially simplifies the system to behave as if it were a single object of mass μ.

Once the reduced mass is calculated, the frequency of oscillation (ω) for a spring-mass system is determined by the formula:

$$\omega = \sqrt{\frac{k}{\mu}}$$

Substituting the expression for reduced mass into the formula, we get:

$$\omega = \sqrt{k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$$

This expression gives the angular frequency of the simple harmonic motion for the system, where k is the spring constant. This encapsulates how the two masses and the spring constant interact to influence the oscillation frequency.

Question19

A mass M, attached to a horizontal spring executes simple harmonic motion with amplitude A_1. When mass M passes mean position, then a smaller mass millis attached to it and both of them together executing simple harmonic motion with amplitude A_2. Then, value of $\frac{A_1}{A_2}$ is

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. $\sqrt{\frac{m^2+M^2}{M^2}}$

B. $\sqrt{\frac{m+M}{M^2}}$

C. $\sqrt{\frac{m+M}{M}}$

D. $\frac{m+M}{M}$

Answer: C

Solution:

To find the ratio of amplitudes, $\frac{A_1}{A_2}$, for the simple harmonic motion (SHM) described, we need to consider the changes when a mass m is added to a mass M, which is attached to a spring.

Key Equations:

Time Period for Mass $(M + m)$:

$$T_2 = 2\pi\sqrt{\frac{m+M}{k}}$$

Time Period for Mass M Only:

$$T_1 = 2\pi\sqrt{\frac{M}{k}}$$

Velocity Considerations:

Assume v_1 is the velocity of the mass M at the mean position and v_2 is the velocity of the combined mass $(m + M)$.

Using Conservation of Linear Momentum:

$$Mv_1 = (m + M)v_2$$

Since $v_1 = A_1\omega_1$ and $v_2 = A_2\omega_2$:

$$M(A_1\omega_1) = (m + M)(A_2\omega_2)$$

Rewriting for the amplitude ratio $\frac{A_1}{A_2}$:

$$\frac{A_1}{A_2} = \frac{(m+M)}{M} \times \frac{\omega_2}{\omega_1}$$

Given $\omega_1 = \frac{2\pi}{T_1}$ and $\omega_2 = \frac{2\pi}{T_2}$, we have:

$$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2}$$

Substituting Time Periods:

Substituting from earlier equations:

$$\frac{T_1}{T_2} = \sqrt{\frac{M}{m+M}}$$

Therefore:

$$\frac{A_1}{A_2} = \frac{(m+M)}{M} \times \sqrt{\frac{M}{m+M}}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$$

This provides the required ratio of the amplitudes when a smaller mass is attached, leading to a new simple harmonic motion with the described properties.

Question20

The displacement of a particle of mass 2 g executing simple harmonic motion is $x=8\cos\left(50t+\frac{\pi}{12}\right)\mathrm{m}$, where t is time in second. The maximum kinetic energy of the particle is

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. 160 J

B. 80 J

C. 40 J

D. 20 J

Answer: A

Solution:

The particle has a mass of:

$m=2\,\mathrm{g}=2\times10^{-3}\,\mathrm{kg}$

The displacement of the particle is described by:

$x=8\cos\left(50t+\frac{\pi}{12}\right)\mathrm{m}$

The maximum velocity v_{max} can be determined using the formula:

$v_{\text{max}}=A\omega$

where $A=8\,\mathrm{m}$ and $\omega=50\,\mathrm{rad/s}$. Thus:

$v_{\text{max}}=8\times50=400\,\mathrm{m/s}$

The maximum kinetic energy (KE_{max}) is given by:

$\mathrm{KE}_{\text{max}}=\frac{1}{2}m\left(v_{\text{max}}\right)^2$

Substituting the known values:

$\mathrm{KE}_{\text{max}}=\frac{1}{2}\times2\times10^{-3}\times\left(4\times10^2\right)^2$

$=10^{-3}\times16\times10^4$

Therefore, the maximum kinetic energy is:

$KE_{max} = 160\,J$

Question21

The relation between the force (F in Newton) acting on a particle executing simple harmonic motion and the displacement of the particle (y in metre) is $500F + \pi^2 y = 0$. If the mass of the particle is 2 g . The time period of oscillation of the particle is

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. 8 s

B. 6 s

C. 2 s

D. 4 s

Answer: C

Solution:

Given:

Mass of the particle: $m = 2\,g = 2 \times 10^{-3}\,kg$

Displacement-force relationship: $500F + \pi^2 y = 0$

From this equation, we can express the force F in terms of the displacement y:

$F = \frac{-\pi^2 y}{500}$

In simple harmonic motion (SHM), force is also given by:

$F = -ky$

By comparing the two force equations, we identify the spring constant k:

$k = \frac{\pi^2}{500}$

The angular frequency ω for SHM is determined by:

$\omega = \sqrt{\frac{k}{m}}$

Substituting the values for k and m:

$\omega = \sqrt{\frac{\pi^2}{500 \times 2 \times 10^{-3}}} = \pi \text{ rad/s}$

The time period T of SHM is given by:

$T = \frac{2\pi}{\omega}$

Plugging in the value of ω:

$T = \frac{2\pi}{\pi} \Rightarrow T = 2 \text{ seconds}$

Therefore, the time period of oscillation of the particle is 2 seconds.

Question22

Two simple harmonic motions are represented by $y_1 = 5[\sin 2\pi t + \sqrt{3} \cos 2\pi t]$ and $y_2 = 5 \sin \left[2\pi t + \frac{\pi}{4}\right]$. The ratio of their amplitudes is

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. 1 : 1

B. 2 : 1

C. 1 : 3

D. $\sqrt{3}$: 1

Answer: B

Solution:

Here, given

$$y_1 = 5[\sin 2\pi t + \sqrt{3}\cos 2\pi t]$$
$$y_2 = 5\sin\left[2\pi t + \frac{\pi}{4}\right]$$

For $y_1 = 10\left[\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t\right]$

$$= 10\left(\cos\frac{\pi}{3}\sin 2\pi t + \sin\frac{\pi}{3}\cos 2\pi t\right)$$
$$y_1 = 10\left[\sin 2\pi t + \frac{\pi}{3}\right]$$

Amplitude in y_1 is $A_1 = 10$

Similarly, $y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$

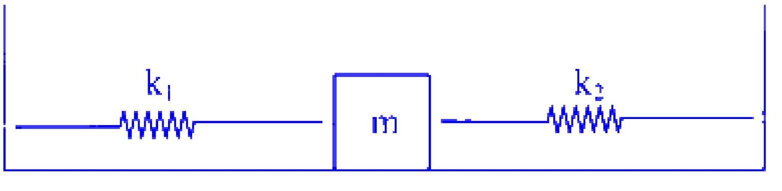
Amplitude in y_2 is $A_2 = 5$

Thus, $\frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$

The ratio of their amplitude 2 : 1.

Question23

When a mass m is connected individually to the springs k_1 and k_2, the oscillation frequencies are v_1 and v_2. If the same mass is attached to the two springs as shown in the figure, the oscillation frequency would be



AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $v_1 + v_2$

B. $\sqrt{v_1^2 + {v_2}^2}$

C. $\left(\frac{1}{v_1}+\frac{1}{v_2}\right)^{-1}$

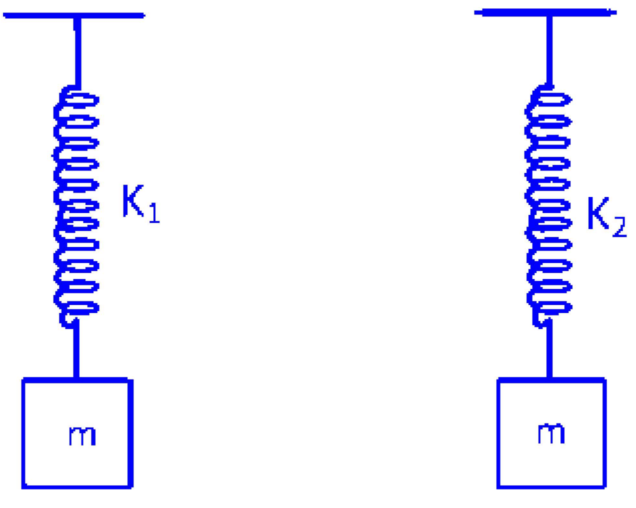
D. $\sqrt{{v_1}^2-{v_2}^2}$

Answer: B

Solution:

When the mass is connected to the two spring individually.

$$v_1=\frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \qquad \ldots(i)$$
$$v_2=\frac{1}{2\pi}\sqrt{\frac{k_2}{m}} \qquad \ldots(ii)$$



Now, the block is connected with two springs, $k_{\text{eq}} = k_1+k_2$

Time period, $T=2\pi\sqrt{\frac{m}{k_{\text{eq}}}}$

$=2\pi\sqrt{\frac{m}{k_1+k_2}}$

$$\text{Frequency } =\frac{1}{T}=\frac{1}{2\pi}\times\sqrt{\frac{k_1+k_2}{m}} \qquad \ldots(iii)$$
$$v=\frac{1}{2\pi}\left[\frac{k_1}{m}+\frac{k_2}{m}\right]^{1/2}$$

$$\text{From Eq. (i) } \frac{k_1}{m}=4\pi^2 v_1^2 \text{ and } \frac{k_2}{m}=4\pi v_2^2$$
$$v=\frac{1}{2\pi}\left[\frac{4\pi^2 v_1^2}{1}+\frac{4\pi^2 v_2^2}{1}\right]^{1/2}$$
$$=\frac{2\pi}{2\pi}\left[v_1^2+v_2^2\right]^{1/2}$$
$$v=\sqrt{v_1^2+v_2^2}$$

Question24

One bar magnet is in simple harmonic motion with time period T in an earth's magnetic field. If its mass is increased by 9 times the time period becomes

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $3T$

B. $9T$

C. $4T$

D. $\sqrt{3}T$

Answer: A

Solution:

We start with the formula for the time period of a magnetic dipole in a magnetic field:

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad ...(i)$$

where I is the moment of inertia, M is the magnetic moment, and B is the magnetic field.

Since the moment of inertia I is directly proportional to the mass m of the magnet—meaning $I \propto m$—if the mass is increased by 9 times, then:

$$I' = 9I$$

The new time period T' can be expressed as:

$$T' = 2\pi\sqrt{\frac{9I}{MB}} \quad ...(ii)$$

By dividing equation (ii) by equation (i), we find:

$$\frac{T'}{T} = \frac{2\pi\sqrt{\frac{9I}{MB}}}{2\pi\sqrt{\frac{I}{MB}}}$$

Simplifying gives:

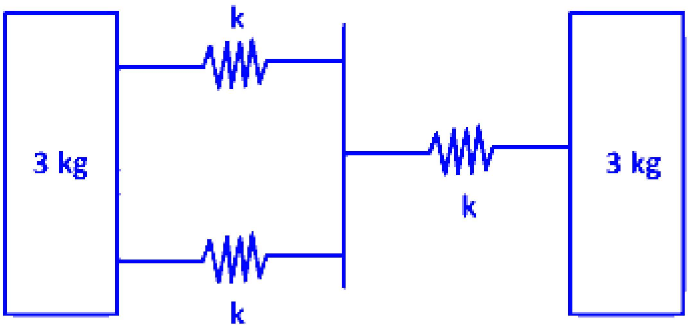
$\frac{T'}{T} = \sqrt{9} = 3$

Therefore, the new time period T' is three times the original time period:

$T' = 3T$

Question25

In a spring block system as shown in figure. If the spring constant $k = 9\pi^2 \mathrm{Nm}^{-1}$, then the time period of oscillation is



AP EAPCET 2024 - 21th May Evening Shift

Options:

A. 1 s

B. 3.14 s

C. 1.414 s

D. 0.5 s

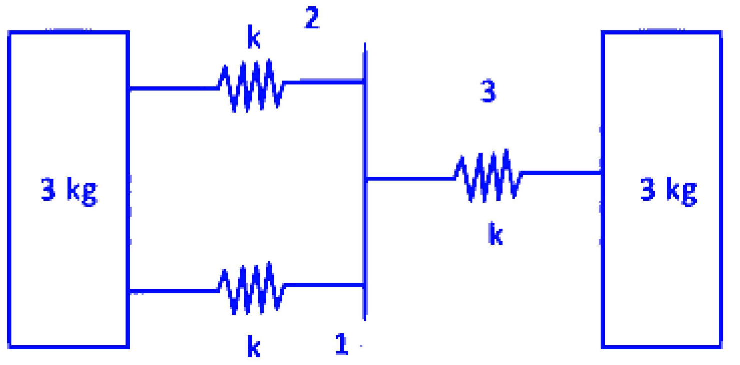
Answer: A

Solution:

Given, $k = 9\pi^2$ N/m

Reduced mass of system is

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$
$$\Rightarrow \quad \mu = \frac{3 \times 3}{3 + 3} = \frac{3}{2} \text{ kg} \qquad \ldots(i)$$



Now, parallel combination of spring constant (between (1) and (2))

$k_{\text{eq}} = k + k = 2k$

Now, k_{eq} and (3) are in series connection,

$$\begin{aligned} k'_{\text{eq}} &= \frac{2k \times k}{2k + k} = \frac{2}{3}k \qquad \ldots (ii) \\ &= \frac{2}{3} \times 9\pi^2 \\ k'_{\text{eq}} &= 6\pi^2 \end{aligned}$$

Time period is given by formula,

$$\begin{aligned} T &= 2\pi\sqrt{\frac{\mu}{k}} \\ &= 2\pi\sqrt{\frac{3}{2} \times \frac{1}{6\pi^2}} = 1 \text{ s} \end{aligned}$$

Question26

A body is executing simple harmonic motion. At a displacement x its potential energy is E_1 and at a displacement y its potential energy is E_2. The potential energy E at a displacement $(x + y)$ is

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. $\sqrt{E} = \sqrt{E_1} - \sqrt{E_2}$

B. $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$

C. $E = E_1 - E_2$

D. $E + E_1 + E_2$

Answer: B

Solution:

Given the scenario where a body is undergoing simple harmonic motion:

The potential energy at displacement x is denoted as E_1.

Since the body is in simple harmonic motion, the relation is given by:

$E_1 = \frac{1}{2} m\omega^2 x^2$

From this, we can derive:

$\sqrt{E_1} = x\sqrt{\frac{1}{2} m\omega^2}$ ...(i)

At displacement y, the potential energy is E_2.

Similarly:

$E_2 = \frac{1}{2} m\omega^2 y^2$

Leading to:

$\sqrt{E_2} = y\sqrt{\frac{1}{2} m\omega^2}$ ...(ii)

For displacement $(x + y)$, the potential energy is denoted by E.

It can be expressed as:

$E = \frac{1}{2} m\omega^2 (x + y)^2$

Therefore:

$\sqrt{E} = (x + y)\sqrt{\frac{1}{2} m\omega^2} = x\sqrt{\frac{1}{2} m\omega^2} + y\sqrt{\frac{1}{2} m\omega^2}$

By using equations (i) and (ii), we find:

$\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$

Question27

The mass of a particle is 1 kg and it is moving along X-axis. The period of its oscillation is $\frac{\pi}{2}$. Its potential energy at a displacement of 0.2 m is

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. 0.24 J

B. 0.48 J

C. 0.32 J

D. 0.16 J

Answer: C

Solution:

Given:

Mass, $m = 1\,\text{kg}$

Displacement, $x = 0.2\,\text{m}$

Period, $T = \frac{\pi}{2}$

We need to find the potential energy at this displacement.

First, we recall the formula for the period of oscillation:

$T = \frac{2\pi}{\omega}$

The angular frequency ω is related to the mass m and the spring constant k by:

$\omega = \sqrt{\frac{k}{m}}$

Substituting this expression for ω into the period equation, we have:

$T = \frac{2\pi \cdot \sqrt{m}}{\sqrt{k}}$

Given $T = \frac{\pi}{2}$, we can solve for k:

$\frac{\pi}{2} = \frac{2\pi \times \sqrt{1}}{\sqrt{k}}$

Solving for k:

$\frac{\pi}{2} = \frac{2\pi}{\sqrt{k}}$

$\sqrt{k} = 4$

$k = 16\,\text{N/m}$

Now, compute the potential energy U at the displacement $x = 0.2\,\text{m}$ using:

$U = \frac{1}{2}kx^2$

Substitute the values:

$U = \frac{1}{2} \times 16 \times (0.2)^2$

$U = 8 \times 0.04$

$U = 0.32\,\text{J}$

Therefore, the potential energy at a displacement of 0.2 m is 0.32 J.

Question28

The potential energy of a particle of mass 10 g as a function of displacement x is $\left(50x^2 + 100\right)$J. The frequency of oscillation is

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. $\frac{10}{\pi} s^{-1}$

B. $\frac{5}{\pi}\ \text{s}^{-1}$

C. $\frac{100}{\pi}\ \text{s}^{-1}$

D. $\frac{50}{\pi}\ \text{s}^{-1}$

Answer: D

Solution:

Given a particle with mass $m = 10\,\text{g} = 0.01\,\text{kg}$, the potential energy as a function of displacement x is expressed as:

$U(x) = 50x^2 + 100$

We recognize that the potential energy for a harmonic oscillator is given by:

$U(x) = \frac{1}{2}kx^2$

To find the spring constant k, we compare the two expressions:

$\frac{1}{2}kx^2 = 50x^2$

Solving for k:

$\frac{1}{2}k = 50k = 100$

The frequency of oscillation f is derived from the angular frequency ω, given by:

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \cdot \sqrt{\frac{k}{m}}$

Substituting the values for k and m:

$f = \frac{1}{2\pi} \cdot \sqrt{\frac{100}{0.01}} f = \frac{100}{2\pi} f = \frac{50}{\pi} \text{ s}^{-1}$

Question29

A horizontal board is performing simple harmonic oscillations horizontally with an amplitude 0.3 m and a period of 4 s . The minimum coefficient of friction between a heavy body placed on the board if the body does not slip will be

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. $\mu = 0.05$

B. $\mu = 0.075$

C. $\mu = 0.173$

D. $\mu = 1.14$

Answer: B

Solution:

To find the minimum coefficient of friction (μ) required for a heavy body to remain stationary on a horizontally oscillating board, we start with the given amplitude and period:

Amplitude (A) = 0.3 m

Period (T) = 4 s

The force associated with an oscillating system is given by $F = kx$. For the body not to slip due to the board's oscillations, the frictional force (f) must be equal to the force due to the oscillation. Thus, we have:

$f = \mu N = \mu mg$

Replacing the spring force expression with terms for mass (m) and angular frequency (ω), we have:

$\mu mg = m\omega^2 A$

Simplifying this expression to solve for μ:

$\mu = \frac{\omega^2 A}{g}$

Knowing that the angular frequency ω can be expressed in terms of the period:

$\omega = \frac{2\pi}{T}$

We substitute ω to find μ:

$\mu = \left(\frac{2\pi}{T}\right)^2 \frac{A}{g}$

Substituting the provided values:

$\mu = \left(\frac{2\pi}{4}\right)^2 \frac{0.3}{9.8}$

Calculate μ:

$\mu = \frac{4\times\pi^2\times0.3}{4\times4\times9.8}$

Finally, compute the numerical value:

$\mu = 0.075$

Thus, the minimum coefficient of friction required is $\mu = 0.075$.

Question30

A test tube of mass 6 g and uniform area of cross-section 10 cm^2 is floating in water vertically when 10 g of mercury is in the bottom. The tube is depressed by a small amount and then released. The time period of oscillation is (acceleration due to gravity $= 10 \text{ ms}^{-2}$)

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. 0.75 s

B. 0.5 s

C. 0.25 s

D. 0.85 s

Answer: C

Solution:

The system consists of a test tube and mercury. The total mass is calculated as:

Total mass (tube + mercury) $= 6 \text{ g} + 10 \text{ g} = 16 \text{ g} = 0.016 \text{ kg}$

The area of the test tube's cross-section is given as:

$A = 10\,\text{cm}^2 = 10^{-3}\,\text{m}^2$

The density of water is:

$\rho = 10^3\,\text{kg/m}^3$

The effective spring constant k due to buoyancy is determined using the formula:

$k = A \times \rho \times g$

Substituting the known values:

$k = 10^{-3} \times 10^3 \times 10 = 10$

Thus, the spring constant k is 10.

The time period of oscillation T is given by:

$T = 2\pi\sqrt{\frac{m}{k}}$

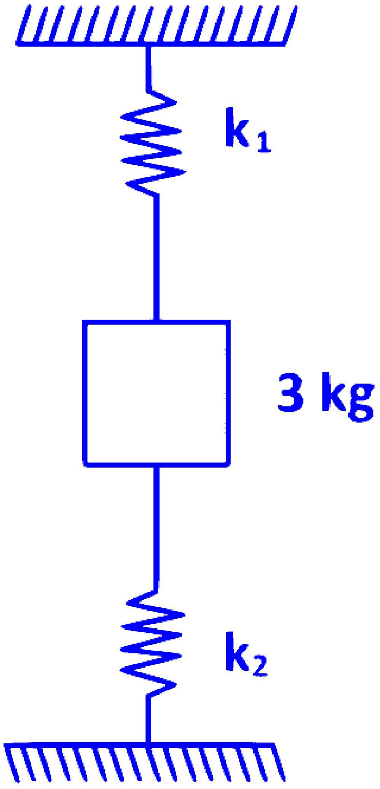
Substituting the mass $m = 0.016\,\text{kg}$ and spring constant $k = 10$:

$T = 2\pi\sqrt{\frac{0.016}{10}} = 0.25\,\text{s}$

Therefore, the time period of oscillation is 0.25 seconds.

Question31

A 3 kg block is connected as shown in the figure. Spring constants of two springs k_1 and k_2 are 50Nm^{-1} and 150Nm^{-1} respectively. The block is released from rest with the springs unstretched. The acceleration of the block in its lowest position is $\left(g = 10\text{ ms}^{-2}\right)$



AP EAPCET 2024 - 20th May Morning Shift

Options:

A. 10 ms^{-2}

B. 12 ms^{-2}

C. 8 ms^{-2}

D. 8.8 ms^{-2}

Answer: A

Solution:

To determine the acceleration of the block at its lowest position, let's start by examining the system's mechanics.

1. Maximum Displacement Calculation

Let x be the maximum downward displacement of the block. The energy conservation equation, considering the force exerted by the springs and gravitational potential energy, is:

$mgx = \frac{1}{2}(k_1 + k_2)x^2$

Substitute the known values:

$x = \frac{2mg}{k_1+k_2} = \frac{2\times3\times10}{50+150} = \frac{60}{200} = 0.3\,\text{m}$

2. Acceleration Calculation at Maximum Displacement

The acceleration of the block at this position, due to the net force from the springs and gravity, is given by:

$a = \frac{(k_1+k_2)x-mg}{m}$

Plug in the calculated values:

$a = \frac{200\times0.3-3\times10}{3} = \frac{60-30}{3} = \frac{30}{3} = 10\,\text{m/s}^2$

Thus, the acceleration of the block at its lowest point is $10\,\text{m/s}^2$.

Question32

In a time of 2 s , the amplitude of a damped oscillator becomes $\frac{1}{e}$ times, its initial amplitude A. In the next two second, the amplitude of the oscillator is

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. $\frac{1}{2\theta}$

B. $\frac{2}{e}$

C. $\frac{1}{e^2}$

D. $\frac{2}{e^2}$

Answer: C

Solution:

The amplitude of a damped oscillator decreases over time due to the damping effect. The relationship for the amplitude is given by:

$A' = Ae^{-\frac{bt}{2m}}$

where A is the initial amplitude, b is the damping coefficient, t is the time, and m is the mass of the oscillator.

Initially, at $t = 2$ seconds, the amplitude becomes $\frac{1}{e}$ of the initial amplitude:

$\frac{A}{e} = Ae^{-\frac{b\times 2}{2m}}$

This simplifies to:

$\frac{A}{e} = \frac{A}{e^{\frac{b}{m}}}$

From this, we deduce:

$\frac{b}{m} = 1$

At $t = 4$ seconds (2 more seconds), the amplitude further becomes:

$A' = Ae^{-\frac{b\times 4}{2m}} = \frac{A}{e^{\frac{4b}{2m}}} = \frac{A}{e^2}$

Thus, the amplitude at $t = 4$ seconds is proportional to $\frac{1}{e^2}$.

Question33

A particle is executing simple harmonic motion with a time period of 3 s . At a position where the displacement of the particle is 60% of its amplitude. The ratio of the kinetic and potential energies of the particle is

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. 5 : 3

B. 16 : 9

C. 4 : 3

D. 25 : 9

Answer: B

Solution:

Given that the time period of the particle executing simple harmonic motion (SHM) is $T = 3\,\text{s}$.

From this, we know:

$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{3}$

Let A represent the maximum amplitude. The displacement of the particle is 60% of its amplitude, which means:

$y = 0.6A$

For SHM, the kinetic energy (KE) is given by:

$\text{KE} = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$

The potential energy (PE) is:

$\text{PE} = \frac{1}{2}m\omega^2 y^2$

The ratio of kinetic energy to potential energy is:

$\frac{\text{KE}}{\text{PE}} = \frac{\frac{1}{2}m\omega^2\left(A^2-y^2\right)}{\frac{1}{2}m\omega^2 y^2} = \frac{A^2-y^2}{y^2}$

Substituting $y = 0.6A$:

$$\begin{aligned}\frac{\text{KE}}{\text{PE}} &= \frac{A^2-(0.6A)^2}{(0.6A)^2} \\ &= \frac{A^2(1-0.36)}{0.36A^2} \\ &= \frac{0.64A^2}{0.36A^2} \\ &= \frac{0.64}{0.36} \\ &= \frac{16}{9}\end{aligned}$$

Thus, the ratio of the kinetic to potential energy when the displacement is 60% of the amplitude is $\frac{16}{9}$.

Question34

The displacement of a particle executing simple harmonic motion is $y = A\sin(2t + \phi)\text{m}$, where t is time in second and ϕ is phase angle. At time $t = 0$, the displacement and velocity of the particle are 2 m

and $4\ \mathrm{ms}^{-1}$. The phase angle, $\phi =$

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. 60°

B. 30°

C. 45°

D. 90°

Answer: C

Solution:

The displacement of a particle undergoing simple harmonic motion is given by $y = A\sin(2t+\phi)$, where t represents time in seconds, and ϕ is the phase angle. At $t = 0$, the displacement is 2 m, and the velocity is $4\ \mathrm{ms}^{-1}$.

Displacement Equation:

$2 = A\sin(2t+\phi) \quad \text{(Equation 1)}$

Velocity Equation:

The velocity v is the derivative of displacement with respect to time:

$v = \frac{dy}{dt} = 2A\cos(2t+\phi)$

At $t = 0$:

$4 = 2A\cos(2t+\phi)$

$\Rightarrow 2 = A\cos(2t+\phi) \quad \text{(Equation 2)}$

Equating Equations (1) and (2):

$A\sin(2t+\phi) = A\cos(2t+\phi)$

Simplifying gives:

$\sin(2t+\phi) = \cos(2t+\phi)$

$\sin(2t+\phi) = \sin\left[90^{\circ} - (2t+\phi)\right]$

This implies:

$2t+\phi = 90^{\circ} - (2t+\phi)$

$4t + 2\phi = 90^{\circ}$

$2t + \phi = 45^\circ$

When $t = 0$:

$\phi = 45^\circ$

Therefore, the phase angle ϕ is 45°.

Question35

The displacement of a damped oscillator is $x(t) = \exp(-0.2t)\cos(3.2t + \phi)$, where t is time in second The time requirement for the amplitude of the oscillator to become $\frac{1}{e^{1.2}}$ times its initial amplitude is

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. 3 s

B. 6 s

C. 2 s

D. 8 s

Answer: B

Solution:

To find the time required for the amplitude of a damped oscillator to become $\frac{1}{e^{1.2}}$ times its initial value, consider the displacement of the oscillator given by:

$x(t) = \exp(-0.2t)\cos(3.2t + \phi)$

Amplitude Analysis

The amplitude of the oscillator is represented as:

$A = e^{-0.2t}$

Initial Amplitude at $t = 0$:

At $t = 0$, the amplitude A_1 is:

$A_1 = e^{-0.2 \times 0} = 1$

Amplitude at $t = t'$:

At time $t = t'$, the amplitude A_2 should be:

$A_2 = \frac{1}{e^{1.2}} A_1 = \frac{1}{e^{1.2}} \times 1 = \frac{1}{e^{1.2}}$

Solving for t'

To find t' such that:

$\frac{1}{e^{1.2}} = e^{-0.2t}$

This relation simplifies to:

$e^{1.2} = e^{0.2t}$

Thus, we have:

$1.2 = 0.2t$

Therefore, solving for t:

$t = \frac{1.2}{0.2} = 6$ seconds

Hence, the time required for the amplitude to reduce to $\frac{1}{e^{1.2}}$ times its initial value is 6 seconds.

Question36

Time period of a simple pendulum in air is T. If the pendulum is in water and executes SHM. Its time period is t. The value of $\frac{T}{t}$ is. (density of bob is $\frac{5000}{3}\ \mathrm{kg\,m}^{-3}$)

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. $\frac{2}{5}$

B. $\sqrt{\frac{2}{5}}$

C. $\frac{5}{2}$

D. $\sqrt{\frac{5}{2}}$

Answer: D

Solution:

Given:

Time period of oscillation of a pendulum in air is T.

Density of the bob, $\rho = \frac{5000}{3}\,\text{kg/m}^3$.

Density of water, $\sigma = 1000\,\text{kg/m}^3$.

We need to find the ratio $\frac{T}{t}$, where t is the time period of the pendulum's oscillation in water.

The time period of oscillation of a pendulum in water can be expressed as:

$$t = \frac{T}{\sqrt{1-\frac{\sigma}{\rho}}}$$

Substitute the given values:

$$t = \frac{T}{\sqrt{1-\frac{1000\times3}{5000}}}$$

This simplifies to:

$$t = \frac{T}{\sqrt{1-\frac{3000}{5000}}}$$

Simplifying further:

$$t = \frac{T}{\sqrt{1-0.6}}$$

$$t = \frac{T}{\sqrt{0.4}}$$

The ratio $\frac{T}{t}$ is:

$$\frac{T}{t} = \sqrt{\frac{1}{0.4}}$$

$$\frac{T}{t} = \sqrt{\frac{5}{2}}$$

Thus, the value of $\frac{T}{t}$ is $\sqrt{\frac{5}{2}}$.

Question37

For a particle executing simple harmonic motion, Match the following statements (conditions) from Column I to statements (shapes of graph) in Columinit

Column I	Column II

a Velocity-displacement graph $(\omega = 1)$	i Straight line
b Acceleration-displacement graph	ii Sinusoidal
c Acceleration - time graph	iii Circle
d Acceleration - velocity $(\omega \neq 1)$	iv Ellipse

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. a-N, b-i, c-il, d-iif

B. $\mathrm{a} - \mathrm{in}, \mathrm{b} - \mathrm{i}, \mathrm{c} - \mathrm{i}, \mathrm{d} = \mathrm{k}$

C. $a = ii, b - 1l, c - 1, d - N$

D. $a - N, b = \bar{c}C = (d - \#$

Answer: B

Solution:

For a body performing simple harmonic motion, the relation between velocity and displacement

$v = \omega\sqrt{A^2 - x^2}$

Square both side

$v^2 = \omega^2\left(A^2 - x^2\right)$

For ($\omega = 1$)

$v^2 + x^2 = A^2$

(a) So, the graph between velocity and displacement will be a circle.

(b) We know that,

$x = A \sin \omega t \quad \ldots$ (i)

Differentiate w.r.t. to t

$\frac{dx}{dt} = v = +\omega A \cos \omega t \quad \ldots$ (ii)

Again differentiate w.r.t. to t,

$\frac{d^2x}{dt^2} = a = -\omega^2 A \sin \omega t \quad \ldots$ (iii)

From Eq. (i),

$a=-\omega^2 x \quad \ldots$ (iv)

Comparing from Eq. $y=mx+c$

It is a equation of straight line with slope, $m=-\omega^2$.

So, the graph between acceleration and displacement will be a straight line.

(c) From Eq. (iii),

$\frac{d^2x}{dt^2}=a=-\omega^2 A\sin\omega t$

The graph between acceleration and time will be a function of sine wave.

(d) From Eq. (A),

$$v=\omega\sqrt{A^2-\omega^2}$$
$$v^2=\omega^2\left(A^2-x^2\right)$$
$$x^2=\left(-\frac{v^2}{\omega^2}+A^2\right)$$
$$x=\sqrt{A^2-\frac{v^2}{\omega^2}}$$

Putting this value in Eq. (iv),

$a=-\omega^2\sqrt{A^2-\frac{v^2}{\omega^2}}$

Square both side

$$a^2=\omega^4\left(A^2-\frac{v^2}{\omega^2}\right)$$
$$a^2=\omega^4 A^2-v^2\omega^2$$
$$\left(\frac{a}{\omega^2}\right)^2=A^2-\frac{v^2}{\omega^2}$$
$$\left(\frac{a}{\omega^2}\right)^2+\left(\frac{v}{\omega}\right)^2=A^2$$

When $\omega\neq 1$, the graph between acceleration a as a function of velocity v in SHM is an ellipse.

Question38

A particle is executing simple harmonic motion with an instantaneous displacement $x=A\sin^2\left(\omega t-\frac{\pi}{4}\right)$. The time period of oscillation of the particle is

Options:

A. $\frac{2\pi}{\omega}$

B. $\frac{\pi}{\omega}$

C. $\frac{\pi}{2\omega}$

D. $\frac{\omega}{2\pi}$

Answer: B

Solution:

Given, instantaneous displacement of particle executing SHM,

$$x = A\sin^2(\omega t - 4)$$

$$\Rightarrow x = A\left[\frac{1-\cos 2(\omega t - 4)}{2}\right]$$

$$\left(\text{Since, } \sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

$$\Rightarrow x = \frac{A}{2}[1-\cos(2\omega t - 8)]$$

Here, angular velocity of particle

$$\omega' = 2\omega$$

$$\Rightarrow \quad \frac{2\pi}{T'} = 2 \times \frac{2\pi}{T} \Rightarrow T' = \frac{T}{2}$$

$$= \frac{\frac{2\pi}{\omega}}{2} = \frac{\pi}{\omega}$$

$$\left(\because T = \frac{2\pi}{\omega}\right)$$

Question39

If the amplitude of a lightly damped oscillator decreases by 1.5% then the mechanical energy of the oscillator lost in each cycle is

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. 1.5%

B. 0.75%

C. 6%

D. 3%

Answer: D

Solution:

The mechanical energy lost in each cycle is proportional to the square of the amplitude A. Therefore, the fractional change in energy can be expressed as:

$$\begin{aligned} \frac{\Delta E}{E} \simeq \frac{dE}{E} &= \frac{dA^2}{A^2} \\ &= \frac{2AdA}{A^2} = 2\frac{dA}{A} \\ &= 2 \times 1.5\% = 3\% \end{aligned}$$

Thus, the mechanical energy of the oscillator lost in each cycle is 3%.

Question40

A body is executing S.H.M. At a displacement x its potential energy is 9 J and at a displacement y its potential energy is 16 J . The potential energy at displacement $(x+y)$ is

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. 25 J

B. 5 J

C. 49 J

D. 7 J

Answer: C

Solution:

According to first condition,

In S. H. M, at displacement x

Potential energy $= 9$ J

-i.e, $\frac{1}{2}kx^2 = 9 \quad \ldots$ (i)

Again, at displacement y,Potential energy

$\frac{1}{2}ky^2 = 16 \quad \ldots$ (ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\frac{1}{2}ky^2}{\frac{1}{2}kx^2} = \frac{16}{9}$$

$$\Rightarrow \quad \left(\frac{y}{x}\right)^2 = \left(\frac{4}{3}\right)^2$$

$$\frac{y}{x} = \frac{4}{3} \quad \ldots \text{(iii)}$$

$\because$ Potential energy of body at displacement $(x+y)$

$$\begin{aligned} \text{PE} &= \frac{1}{2}k(x+y)^2 \\ &= \frac{9}{x^2}(x+y)^2 \quad \text{[From Eq. (i)]} \\ &= 9\frac{(x+y)^2}{x^2} = 9\left(1+\frac{y}{x}\right)^2 \\ &= 9\left(1+\frac{4}{3}\right)^2 \quad \text{[From Eq. (iii)]} \\ &= 49 \text{ J} \end{aligned}$$

Question41

A hydrometer executes simple harmonic motion when it is pushed down vertically in a liquid of density ρ. If the mass of hydrometer is m and the radius of the hydrometer tube is r, then the time period of oscillation is

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. $T=2\pi\sqrt{\frac{m}{\pi^2\rho g}}$

B. $T=2\pi\sqrt{\frac{\pi\pi^2\rho g}{m}}$

C. $T=\frac{1}{2\pi}\sqrt{\frac{m}{\pi\pi^2\rho g}}$

D. $T=\frac{1}{2\pi}\sqrt{\frac{\pi\pi^2\rho g}{m}}$

Answer: A

Solution:

If the hydrometer is in equilibrium, the weight is balanced by the buoyancy force on it by the fluid. When hydrometer liquid is pressed by x distance the net unbalanced force in the upward direction is given by $F=-(\text{Area}\times\text{Density}\times\text{Gravity}\times\text{Distance moved by fluid})$

Where, $-ve$ sign shows that the direction of force will be opposite to the direction of motion.

Thus, $m\frac{d^2x}{dt^2}=-\pi r^2\rho gx$

Were, $r=$ radius of liquid column

$\rho=$ density of liquid

$x=$ distance moved by liquid

Now, $\frac{d^2x}{dt^2}=\frac{-\pi r^2\rho g}{m}x$

or $\omega^2=\frac{\pi r^2\rho g}{m}$ or $\omega=\sqrt{\frac{\pi r^2\rho g}{m}}$

$\Rightarrow\quad T=2\pi\sqrt{\frac{m}{\pi r^2\rho g}}$

Question42

An object undergoing simple harmonic motion takes 0.5 s to travel from one point of zero velocity to the next such point. The angular frequency of the motion is

Options:

A. $\pi \text{ rad s}^{-1}$

B. $2\pi \text{ rad s}^{-1}$

C. $3\pi \text{ rad s}^{-1}$

D. $\frac{\pi}{2} \text{ rad s}^{-1}$

Answer: B

Solution:

The velocity of the object becomes zero at maximum displacement of either side of the mean position in the SHM.

Time taken from one maximum to another maximum $= 0.5 \text{ s}$

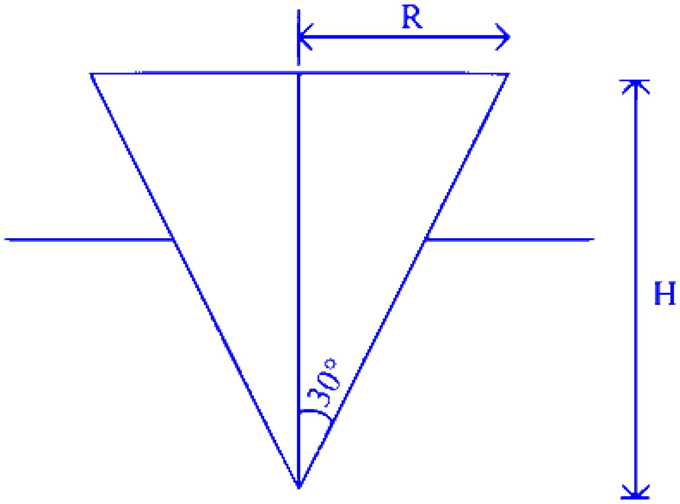
Time period $= 2 \times 0.5 = 1 \text{ s}$

Angular velocity, $\omega = 2\pi v$

$= \frac{2\pi}{T} = \frac{2\pi}{l} = 2\pi \text{ rad/s}$

Question43

A cone with half the density of water is floating in water as shown in figure. It is depressed down by a small distance $\delta(\ll< H)$ and released. The frequency of simple harmonic oscillations of the cone is



AP EAPCET 2022 - 4th July Morning Shift

Options:

A. $\frac{1}{2\pi}\sqrt{\frac{6g}{H}\frac{1}{4^{\frac{1}{3}}}}$

B. $\frac{1}{2\pi}\sqrt{\frac{3g}{H}\frac{1}{4^{\frac{1}{3}}}}$

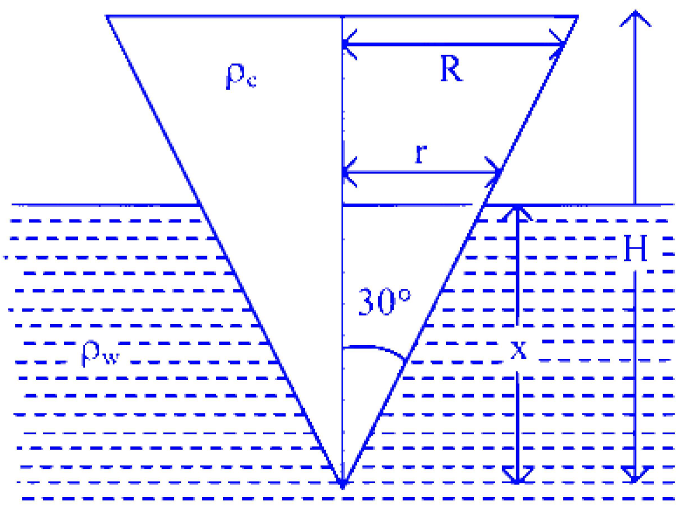
C. $\frac{1}{2\pi}\sqrt{\frac{6g}{2H}}$

D. $\frac{1}{2\pi}\sqrt{\frac{g}{H}}$

Answer: A

Solution:

Let initially the cone be in equilibrium i.e. floating



From the diagram, it is clear that

Buoyant force $(F_b) =$ weight of the cone. (F_w)

$$\rho_w \times\left(\frac{1}{3} \pi r^2 x\right) \times g=\rho_c \times\left(\frac{1}{3} \pi R^2 H\right) \times g$$

$$\Rightarrow \quad 2 r^2 x=R^2 H \quad\left(\because \rho_c=\frac{\rho_w}{2}\right)$$

$$\Rightarrow \quad x=\frac{R^2 H}{2 r^2} \quad \ldots \text{(i)}$$

From the diagram,

$$\tan \theta=\frac{r}{x}=\frac{R}{H}$$

$$\Rightarrow \quad x=\frac{r H}{R} \quad \ldots \text{(ii)}$$

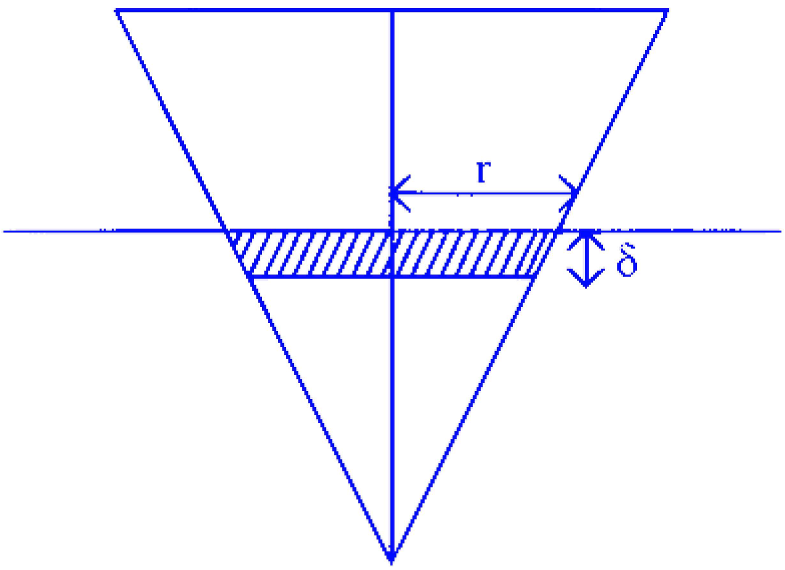
From Eqs. (i) and (ii)

$$\frac{R^2 H}{2 r^2}=\frac{r H}{R}$$

$$\Rightarrow \quad \frac{R^3}{2}=r^3 \Rightarrow r=\left(\frac{1}{2}\right)^{1 / 3} R$$

Now, the cone. is depressed by small distance $\delta(<<H)$ and released it executes simple harmonic motion. In simple harmonic motion, the buoyancy force acts as restoring force which is given as

$F_b=$ density $\times$ volume $\times$ gravity



Since, the displacement is very small, the excess submerged shape of cone. can be considered as a cylinder whose volume is given by $\pi r^2 \delta$.

$$\therefore \quad F_b = \rho_w \pi r^2 \delta g = \rho_w \pi \left[\left(\frac{1}{2}\right)^{1/3} R\right]^2 \delta g$$

Thus, force is equal to restoring force, hence

$$F_b = k\delta = m\omega^2 \delta$$

$$\rho_w \pi \left(\frac{1}{4}\right)^{1/3} R^2 \delta g = \rho_c \frac{1}{3} \pi R^2 H \delta \omega^2$$

$$2 \times 3 \times \left(\frac{1}{4}\right)^{1/3} \delta g = H \delta \omega^2 \quad \left(\because \rho_c = \frac{\rho_w}{2}\right)$$

$$\Rightarrow \quad \frac{6\left(\frac{1}{4}\right)^{1/3} g}{H} = \omega^2$$

$$\text{or} \quad \omega = \sqrt{\frac{6g}{H}\left(\frac{1}{4}\right)^{1/3}} \Rightarrow 2\pi f = \sqrt{\frac{6g}{H}\left(\frac{1}{4}\right)^{1/3}}$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{6g}{H}\left(\frac{1}{4}\right)^{1/3}} = \frac{1}{2\pi}\sqrt{\frac{6g}{H} \cdot \frac{1}{4^{1/3}}}$$

Question44

In case of a forced vibration, the resonance wave becomes very sharp when the

AP EAPCET 2021 - 20th August Evening Shift

Options:

A. quality factor is small

B. damping force is small

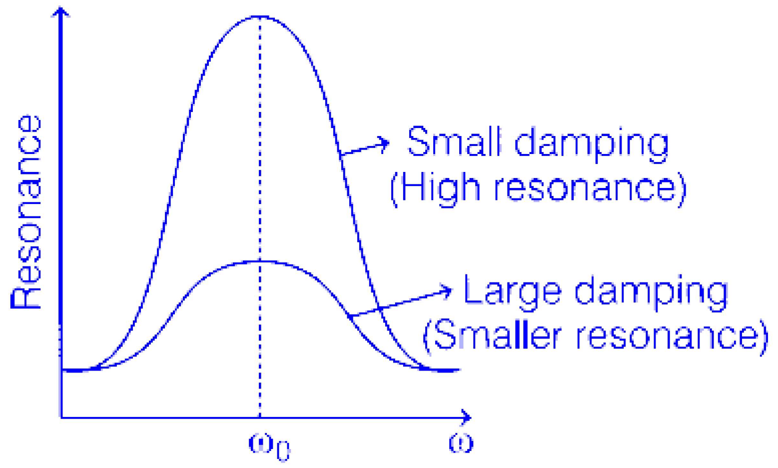
C. restoring force is small

D. applied periodic force is small

Answer: B

Solution:

As we know that, for less damping, resonancepeak is taller and sharper.



Question45

A particle executing simple harmonic motionalong a straight line with an amplitude A,attains maximum potential energy when itsdisplacement from mean position equals

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. 0

B. $\pm\frac{A}{\sqrt{2}}$

C. $\pm A$

D. $\pm\frac{A}{2}$

Answer: C

Solution:

Let, wave displacement be x.

As we know that,

Potential energy, $U = \frac{1}{2}kx^2$

where, k is wave constant.

Potential energy will be maximum, when displacement is maximum.

$\Rightarrow x = \pm A$

Question46

The bob of a simple pendulum is a sphericalhollow ball filled with water. A plugged holenear the bottom of the oscillating bob getssuddenly unplugged. During observation, tillwater is coming out the time period ofoscillation would

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. remain unchanged

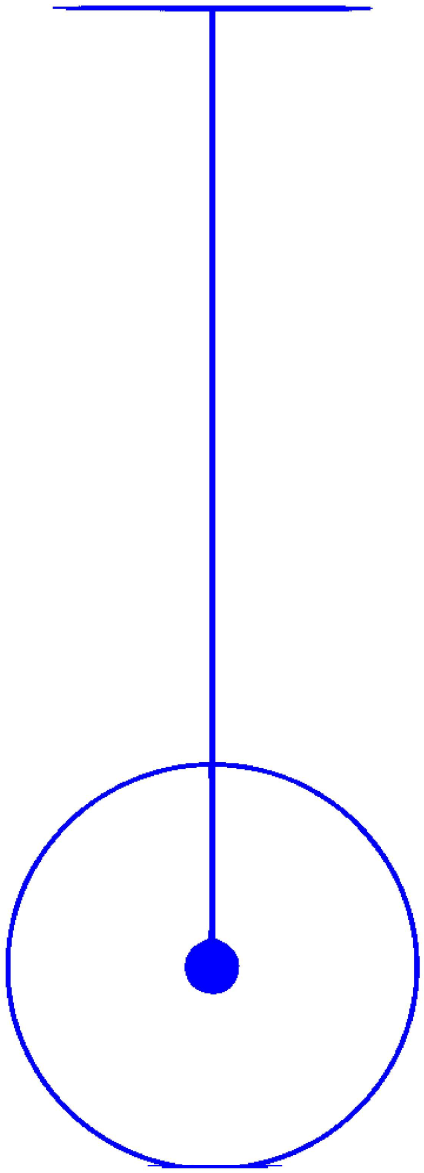
B. increase towards a saturation value

C. first increase and then decrease to the original value

D. first decrease and then increase to the original value

Answer: C

Solution:



Let,

Time period $= T$

Length of simple pendulum $= l$

Acceleration due to gravity $= g$

$\because \quad T = 2\pi\sqrt{l/g}$ .... (i)

When sphere is fully filled with water, centre of mass lies at centre.

Case I For, water level below centre of sphere

$l_1 > l_0$ ...... (ii)

Case II For empty sphere,

$l = l_0 < l_1$ ..... (iii)

$\therefore$ From Eqs. (i), (ii) and (iii)

In case I, time period increase.

In case II, time period decrease.

Question47

A block of mass lkg is fastened to a spring of spring constant of 100 Nm^{-1}. The block is pulled to a distance $x = 10$ cm from its equilibrium position $(x = 0 \text{ cm})$ on a frictionless surface, from rest at $t = 0$. The kinetic energy and the potential energy of the block when it is 5 cm away from the mean position is

AP EAPCET 2021 - 19th August Evening Shift

Options:

A. $0.375 \text{ J}, 0.125 \text{ J}$

B. $0.125 \text{ J}, 0.375 \text{ J}$

C. $0.125 \text{ J}, 0.125 \text{ J}$

D. $0.375 \text{ J}, 0.375 \text{ J}$

Answer: A

Solution:

Given, mass of block, $m = 1$ kg

Spring constant, $k = 100$ N/m

In case I, extension in spring, $\Delta x_1 = 10$ cm

$= 10 \times 10^{-2}$ m

Initial speed, $u = 0$ ms^{-1}

In case II, extension in spring $\Delta x_2 = 5$ cm

$= 5 \times 10^{-2}$ m

Let, kinetic energy and potential energy be (KE) and (PE)

As we know that,

$$\begin{aligned}
\mathrm{KE} &= \frac{1}{2} k\left(\Delta x_1^2 - \Delta x_2^2\right) \\
&= \frac{1}{2} \times 100\left(10^2 - 5^2\right) \times 10^{-4} \\
&= 50 \times 75 \times 10^{-4} = 0.375 \mathrm{~J} \\
\text{and PE} &= \frac{1}{2} k \Delta x_2^2 \\
&= \frac{1}{2} \times 100 \times \left(5 \times 10^{-2}\right)^2 = 50 \times 25 \times 10^{-4} = 0.125 \mathrm{~J}
\end{aligned}$$

Question48

The scale of a spring balance which can measure from 0 to 15 kg is 0.25 m long. If a body suspended from this balance oscillates with a time period $\frac{2\pi}{5}$ s, neglecting the mass of the spring, find the mass of the body suspended.

AP EAPCET 2021 - 19th August Evening Shift

Options:

A. 24 kg

B. 1 kg

C. 20 kg

D. 7 kg

Answer: A

Solution:

Given, range of mass is from m_1 to m_2 i.e. 0 to 15 kg

Length of spring, $x = 0.25$ m

Time period of oscillation, $T = \frac{2\pi}{5}$ s

Let mass of body suspended $= m$

Spring constant $= k$

Acceleration due to gravity $g = 10 \text{ ms}^{-2}$

Since, $T = 2\pi\sqrt{\frac{m}{k}}$ ..... (i)

and $\quad F = mg = kx$

$$\Rightarrow \quad k = \frac{mg}{x} = \frac{15 \times 10}{0.25} \text{ N/m}$$

Substituting in Eq. (i), we get

$$T = 2\pi\sqrt{\frac{m \times 25}{15 \times 10 \times 100}}$$

$$\Rightarrow \quad \left(\frac{2\pi}{5}\right)^2 = (2\pi)^2\left(\frac{m}{15 \times 40}\right)$$

$$\Rightarrow \quad \frac{4\pi^2}{25} = \frac{4\pi^2 m}{15 \times 40} \Rightarrow m = 24 \text{ kg}$$

Question49

A spring is stretched by 0.40 m when a massof 0.6 kg is suspended from it. The period ofoscillations of the spring loaded by 255 g andput to oscillations is close to ($g = 10 \text{ ms}^{-2}$)

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. 11 s

B. 48.6 s

C. 0.82 s

D. 4.86 s

Answer: C

Solution:

Given, expansion of spring, $\Delta x = 0.4$ m

Mass suspended in case **I** (m_1) and case II (m_2) is 0.6 kg and 255 g, respectively.

Acceleration due to gravity $g = 10 \text{ ms}^{-2}$

Since, force $F = mg = k\Delta x$

$$\therefore \text{ Spring constant } k = \frac{mg}{\Delta x}$$
$$\Rightarrow \quad k = \frac{m_1 g}{\Delta x_1} = \frac{0.6 \times 10}{0.4} = 15 \text{ N/m}$$

and time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$$T_2 = 2\pi\sqrt{\frac{m_2}{k}}$$
$$T_2 = 2\pi\sqrt{\frac{0.255}{15}}$$
$$= 2\pi\sqrt{\frac{255}{15 \times 10^3}}$$
$$= 2\pi\sqrt{0.017} = 0.82 \text{ s}$$

Question50

A heavy brass sphere is hung from a springand it executes vertical vibrations with periodT. The sphere is now immersed in anon-viscous liquid with a density (1/10)ththat of brass. When set into verticalvibrations with the sphere remaining insideliquid all the time, the time period will be

Options:

A. $\left(\sqrt{\frac{9}{10}}\right)T$

B. $\left(\sqrt{\frac{10}{9}}\right)T$

C. $\left(\frac{9}{10}\right)T$

D. Unchanged

Answer: D

Solution:

Given, time period of vibration in air = T

Density of fluid = $\rho/10$

Time period of vibration in fluid = T′

Since, $T = 2\pi\sqrt{\frac{m}{k}}$

and as the value of mass (m) and spring constant(k) of spring-mass system is same.

$\therefore$ Time period will remain same.
